Atul Rana

The Field Axioms: A reference framework of coherence

The Complete Mathematics Conference

The field axioms give the mathematician a rules-based approach to working with number and variables.

Knowing and being able to work with the field axioms means that a pupil can attack any arithmetical or algebraic problem they might face in school-level mathematics.

These properties always hold true; they are the same properties at the beginning of primary mathematics as they are at the beginning of calculus.

They hold for whole numbers, fractions, negative numbers, rational numbers, letters and expressions.

From : Mark McCourt
https://emaths.co.uk
Models, metaphors, examples and instruction

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Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
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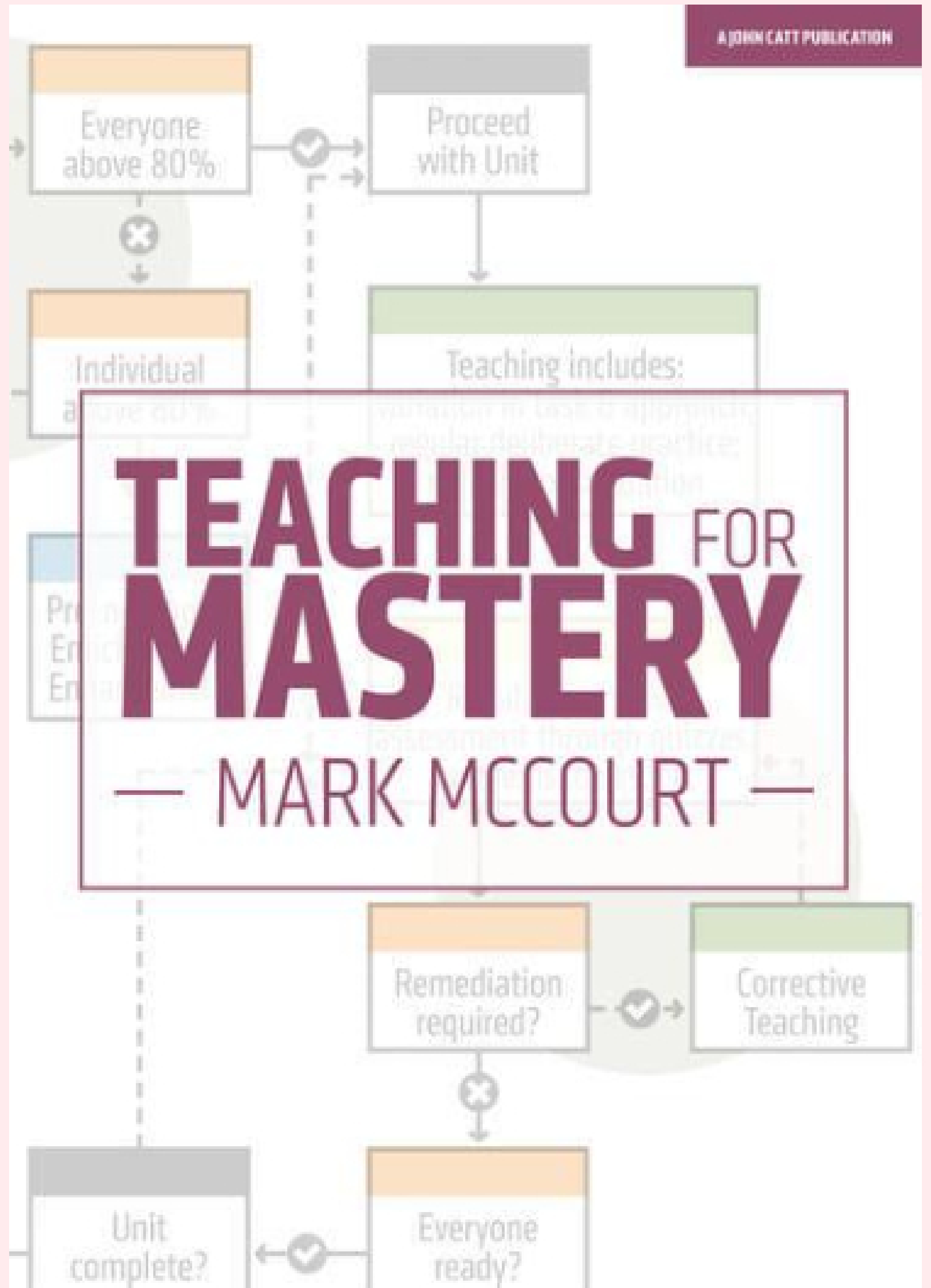
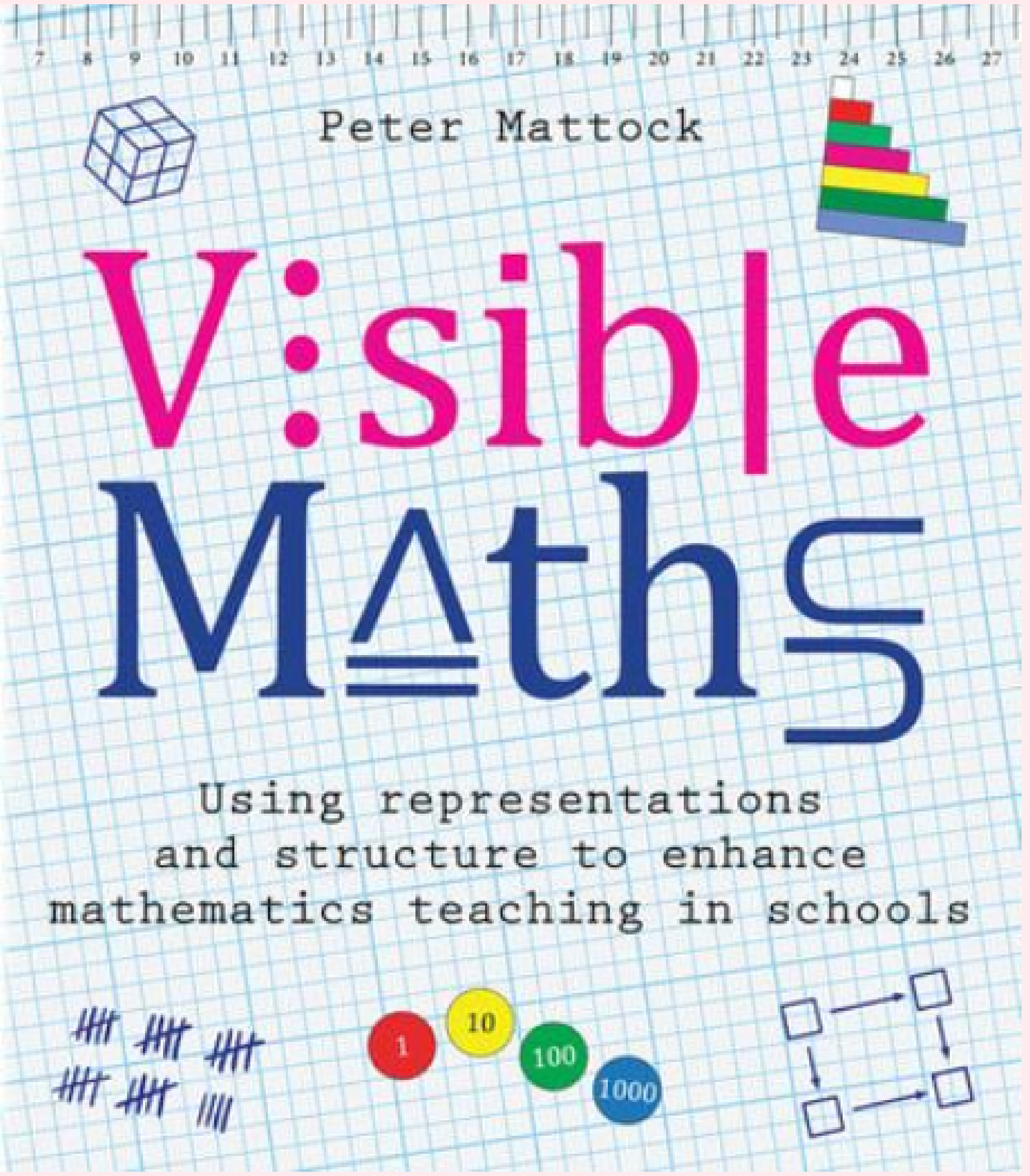




Bernie Westacott
Podcast video
with Mr. Barton

What does axiom mean?

"In mathematics or logic, an axiom is an unprovable rule or first principle accepted as true because it is self-evident or particularly useful."

Etymology for axiom

from the Greek word ἀξίωμα (axíōma)

"to deem worthy", but also "to require", which in turn comes from ἄξιος (áxios), meaning "being in balance"

Wikipedia/Merriam-Webster

$$\begin{array}{r} 3x - 2y = 14 \\ + \; -x + 2y = -10 \\ \hline 2x + 0y = 4 \end{array}$$

(Existence of additive inverses)

$$2x + 0 = 4$$

(Additive identity property of 0)

$$2x = 4$$

$$\frac{1}{2} \times 2x = 4 \times \frac{1}{2}$$

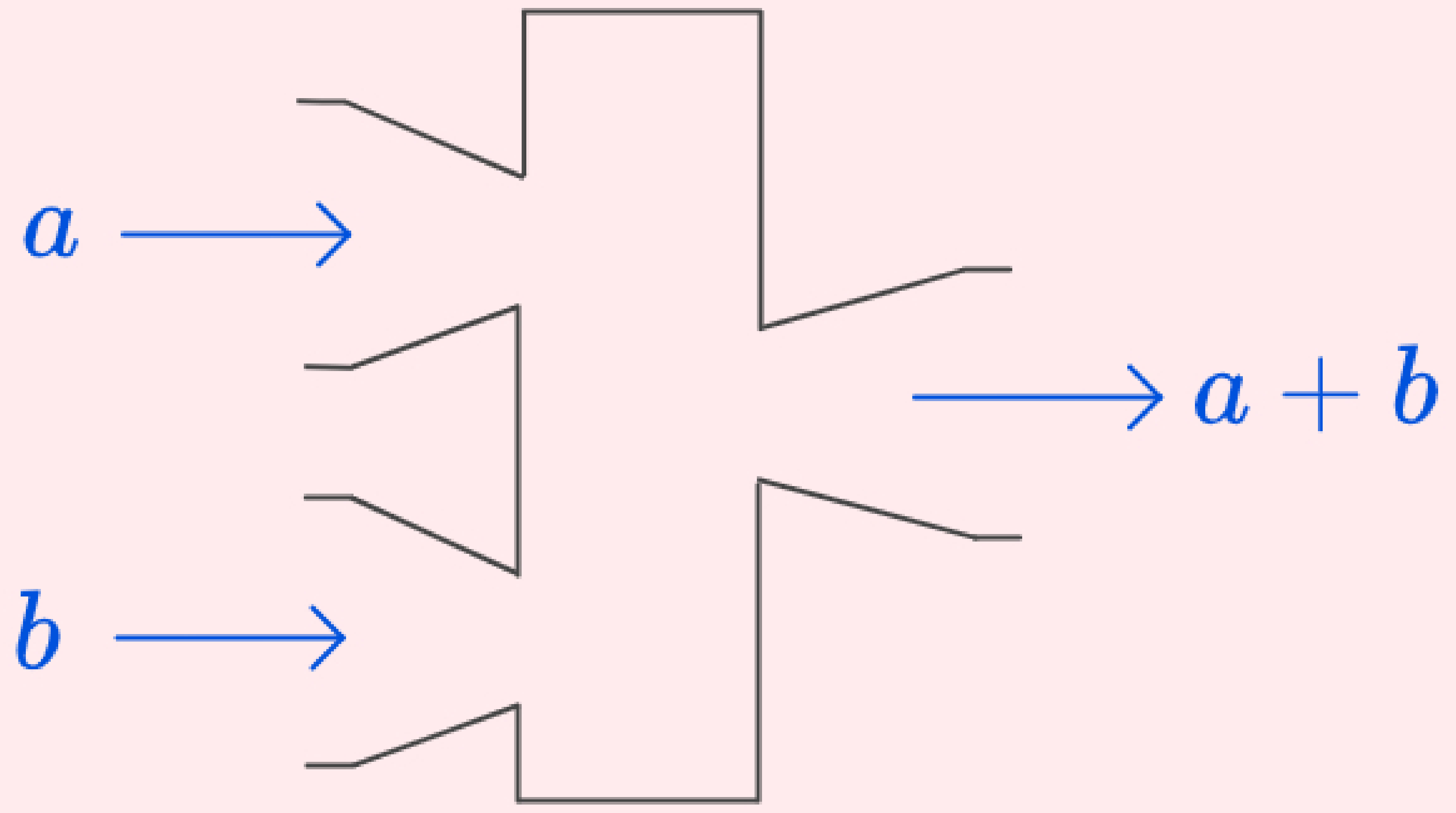
(Equality
Existence of multiplicative inverses)

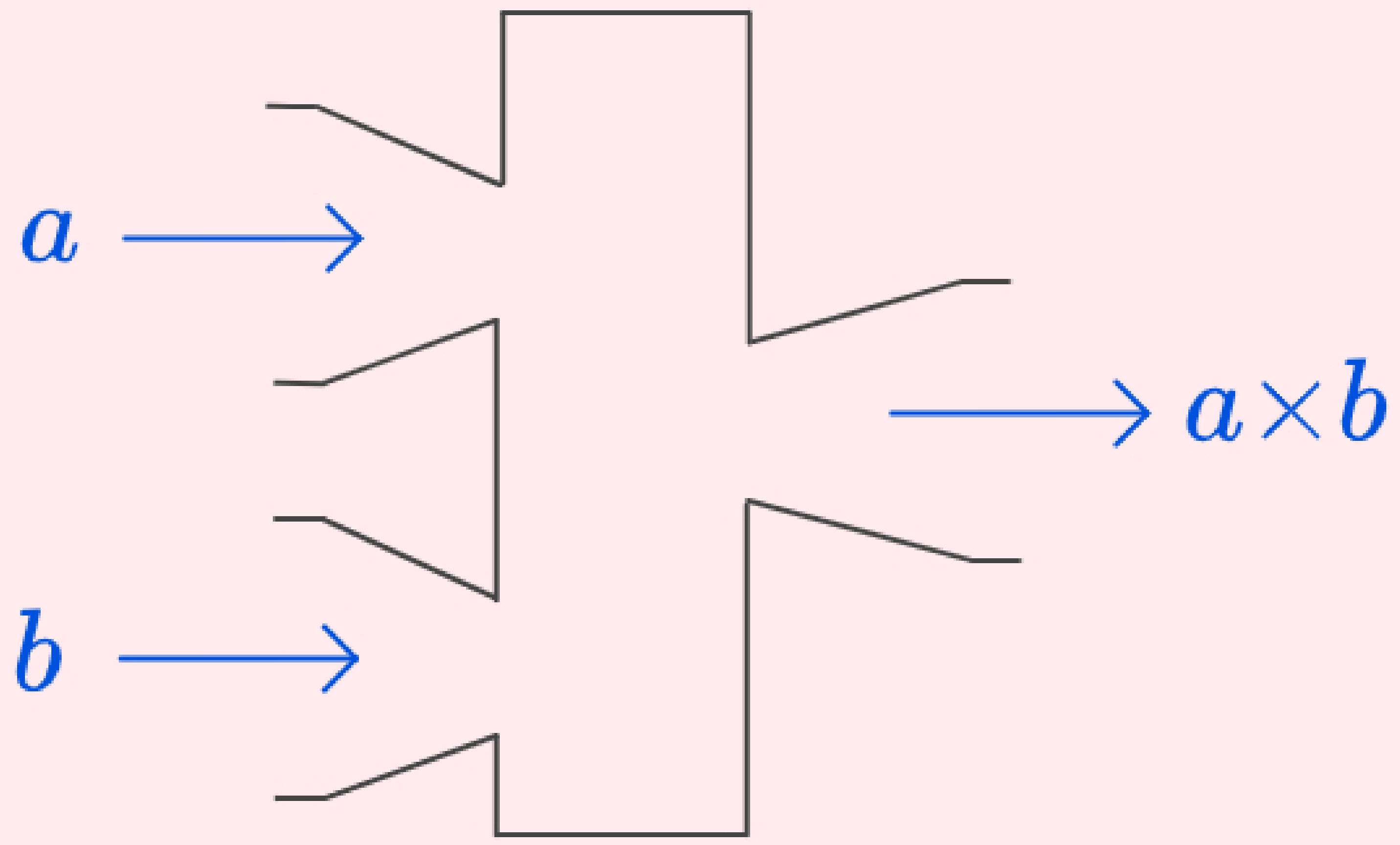
$$1x = 2$$

(Multiplicative identity property of 1)

$$x = 2$$

Binary operation





Equals

The equals symbol, ‘=’, denotes that, for any two given expressions, applying an isomorphic function to each will maintain equality.

Equivalence (relation)

A binary relation that is reflexive, symmetric and transitive. These three conditions must be met:

1. a = a (reflexive property),
2. if a = b then b = a (symmetric property)
3. if a = b and b = c then a = c (transitive property)

Equality

Two expressions have equality if they have the same value or represent the same mathematical object.

Equation

An equation is a mathematical statement asserting that two expressions have equality.

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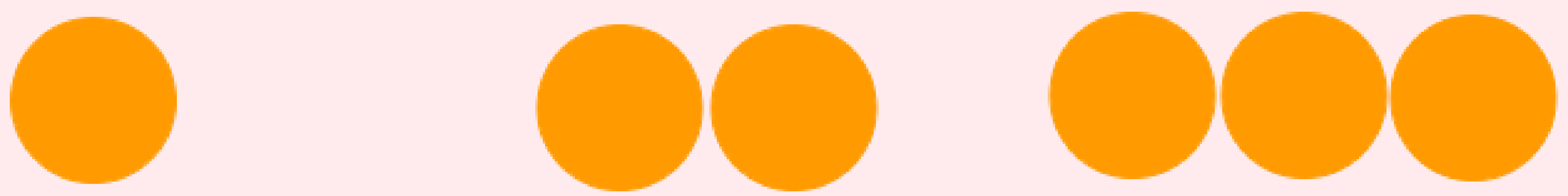
Associative property of addition

To "associate" or group together



$$(1+2)+3=1+(2+3)$$

$$(a+b)+c=a+(b+c)$$

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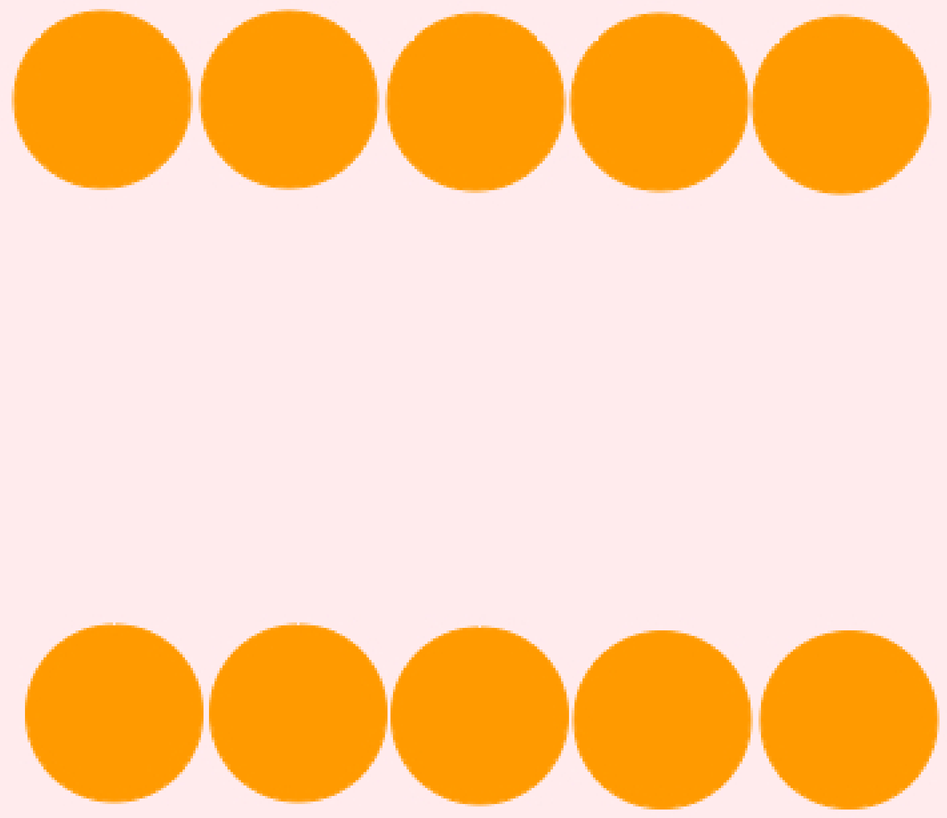
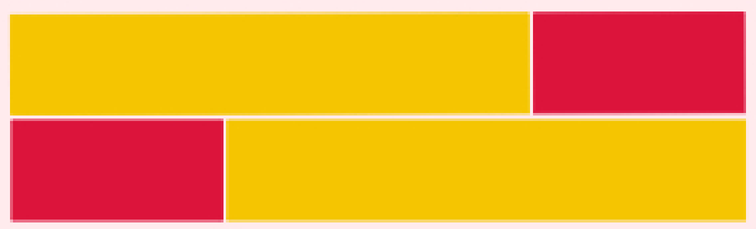
Commutative property of addition

"Commute"



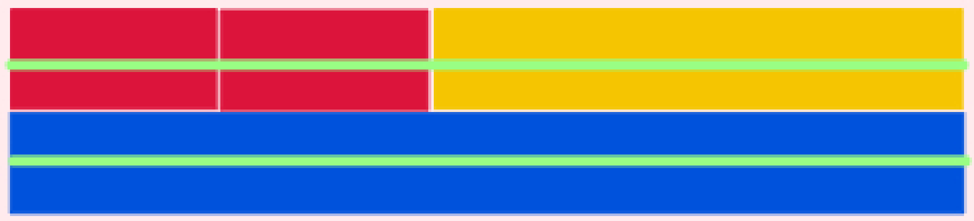
Commutative property of addition

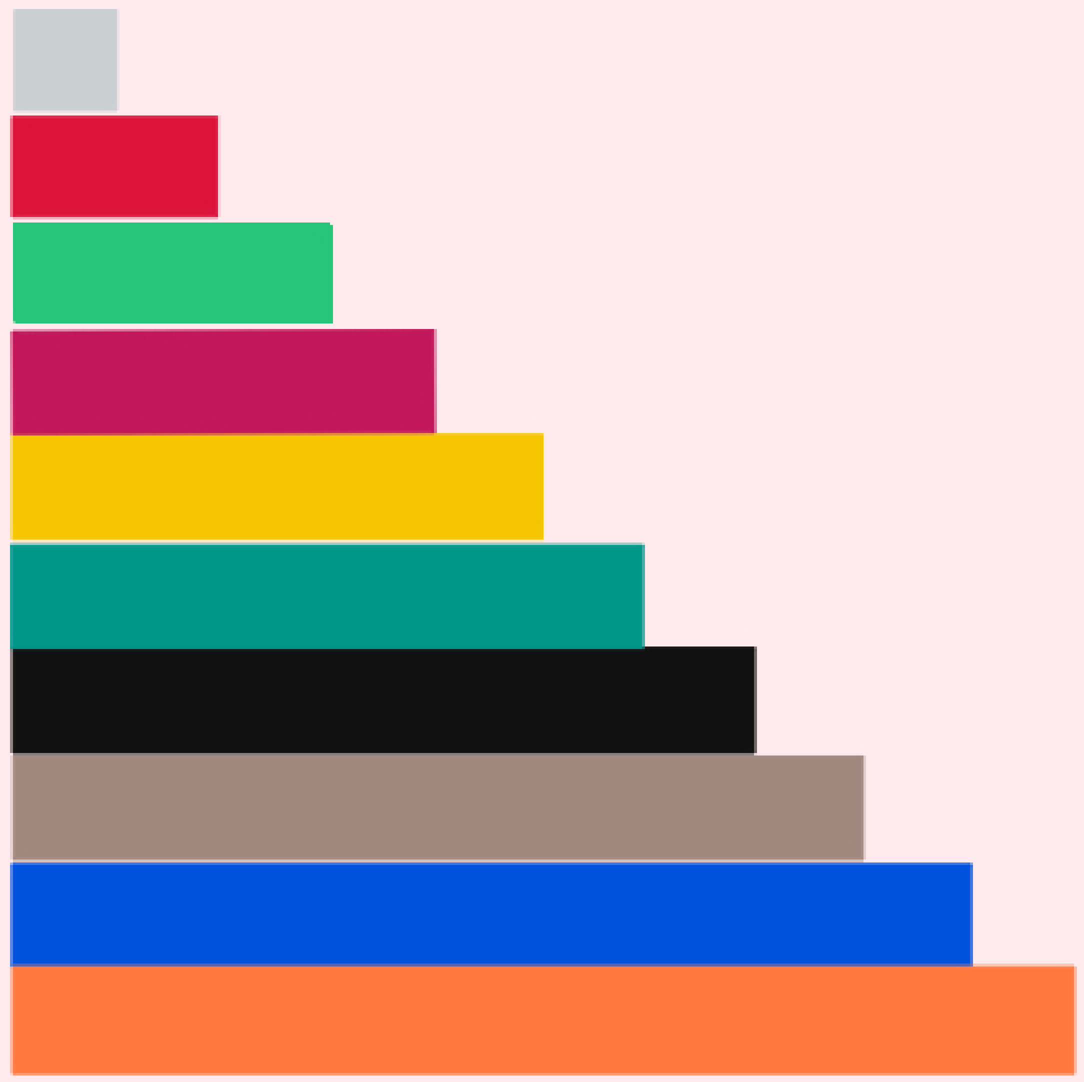
y + r = r + y



r + r + y = bl

2r + y = bl





$$1 + 2 + 3 + 4 = 2 + 3 + 1 + 4$$

$$1 + ({}^{-}2) + 3 + 4 = ({}^{-}2) + 3 + 1 + 4$$

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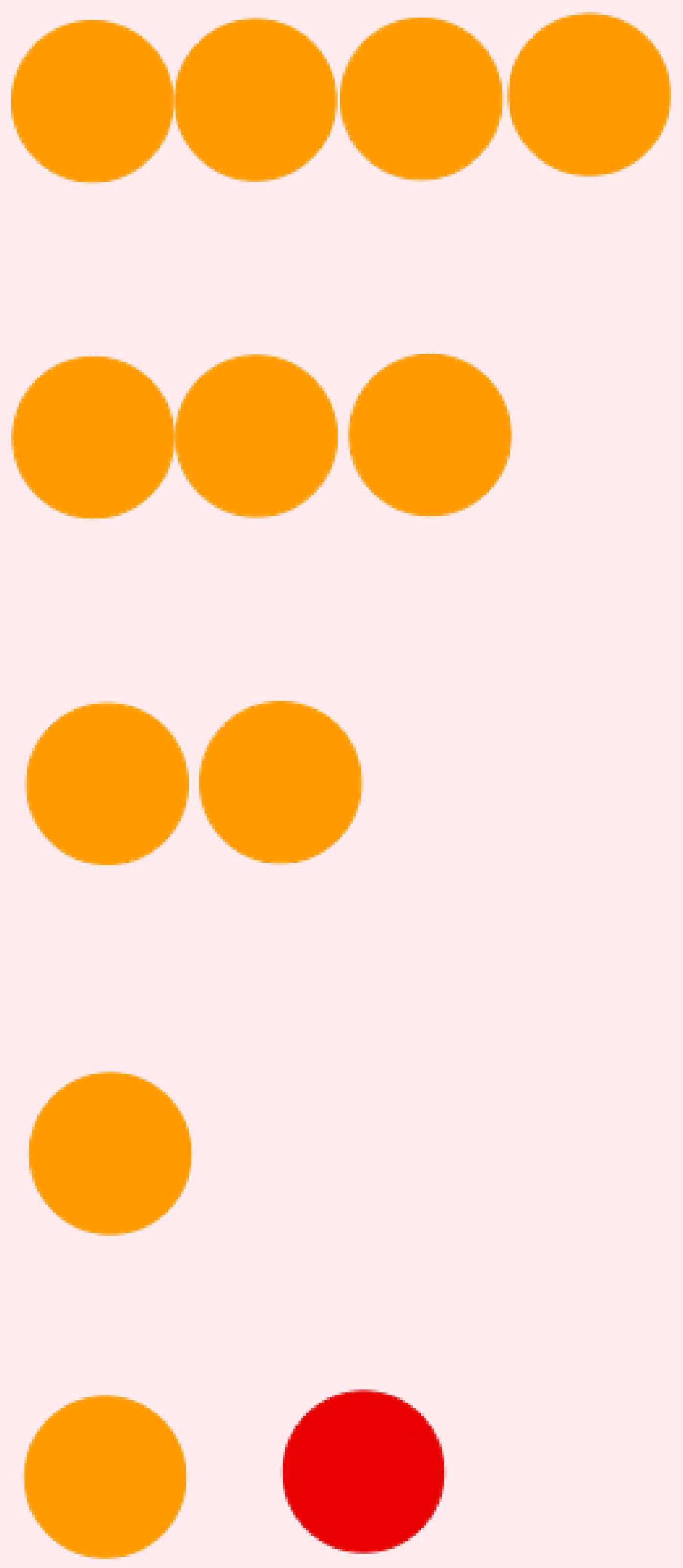


$1 + 1\ 1\ 1 = 4$

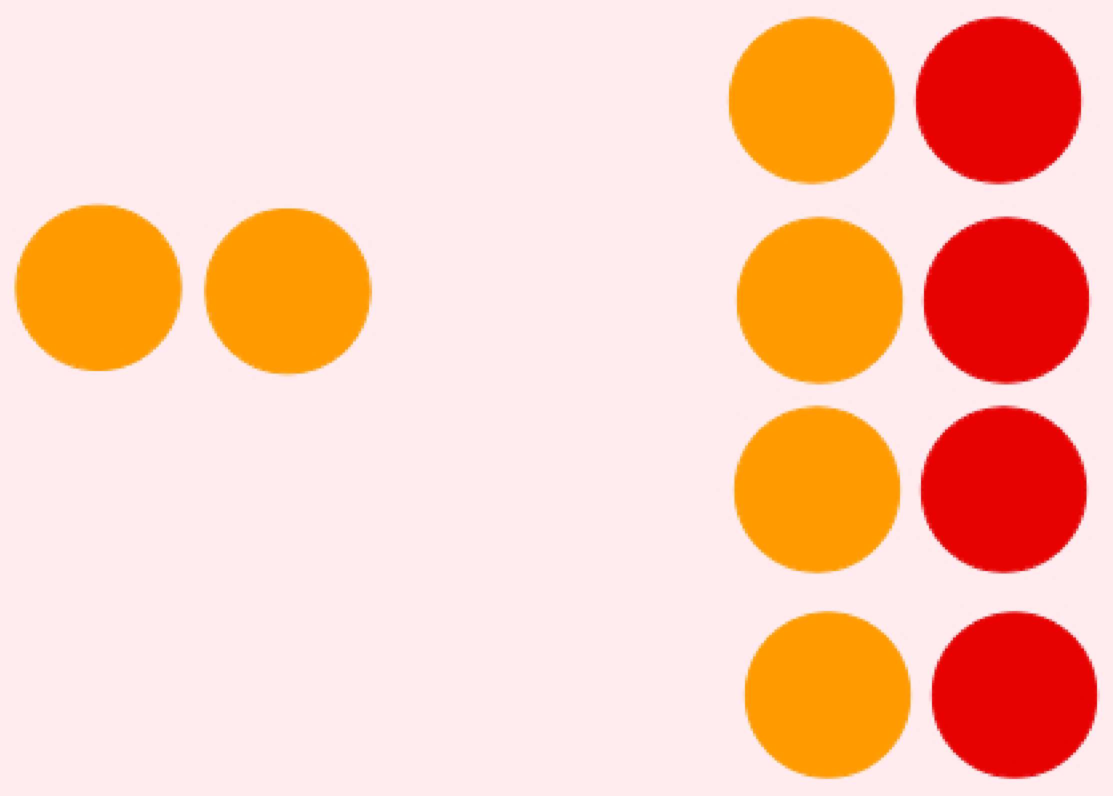
$1 + 1\ 1 = 3$

$1 + 1 = 2$

$1 + \quad = 1$

$1 + {}^{-}1 = 0$

Zero pairs with counters and algebra tiles (mathsbot)



Turning subtraction into addition with additive inverses

$$3 - ({}^{-}2)$$

↓

$$3 + (2)$$

$$3 - (2)$$

↓

$$3 + ({}^{-}2)$$

$$
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+ \quad -x + 2y = -10 \\
\hline
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\end{array}
$$

(Existence of additive inverses)

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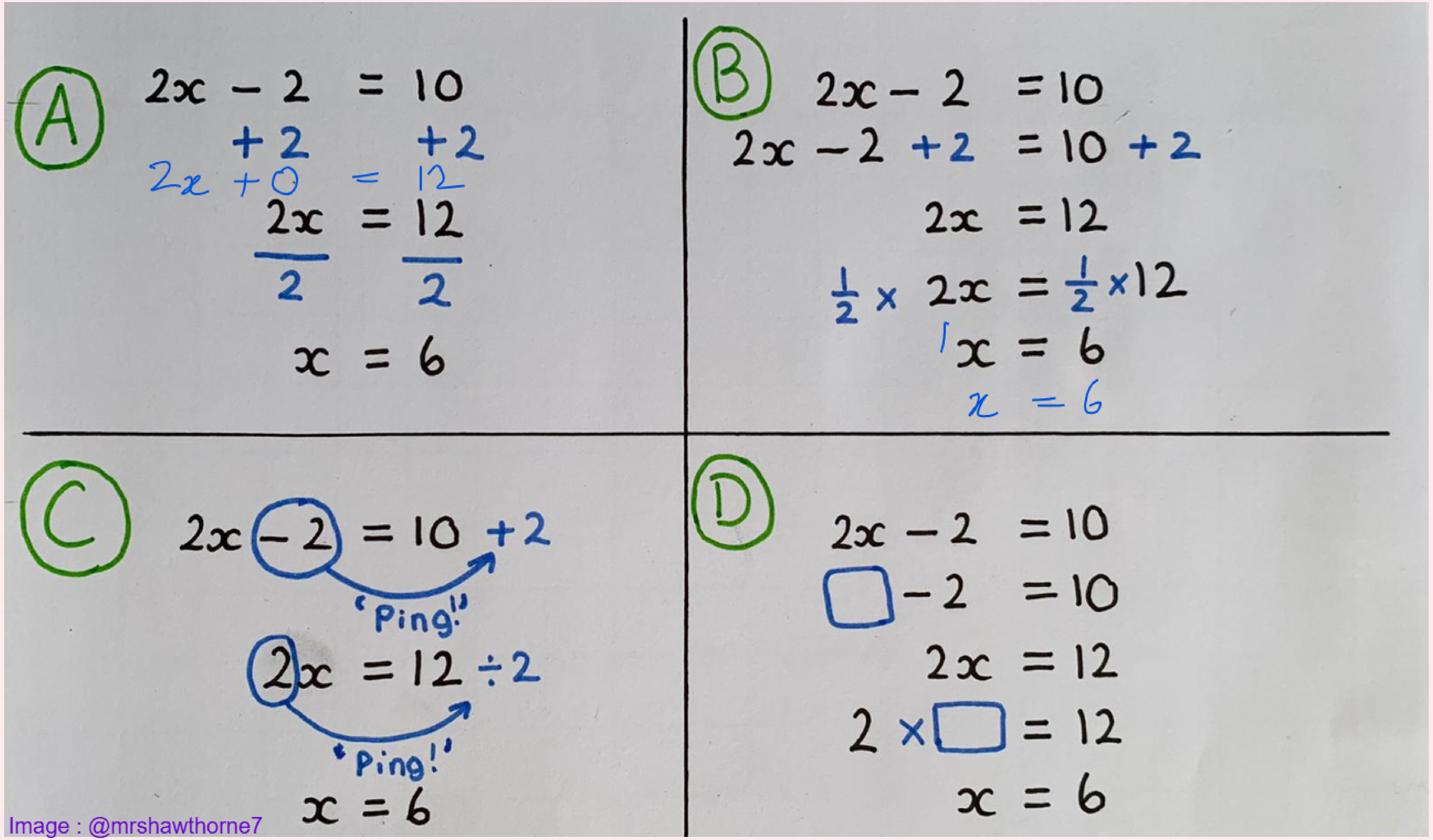

Image : @mrshawthorne7

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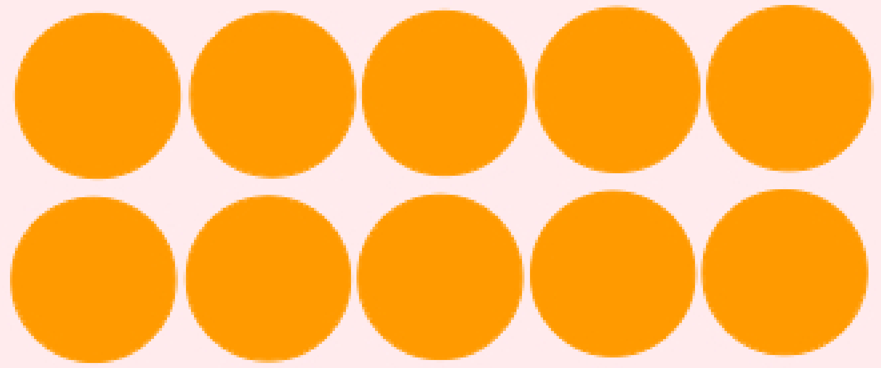
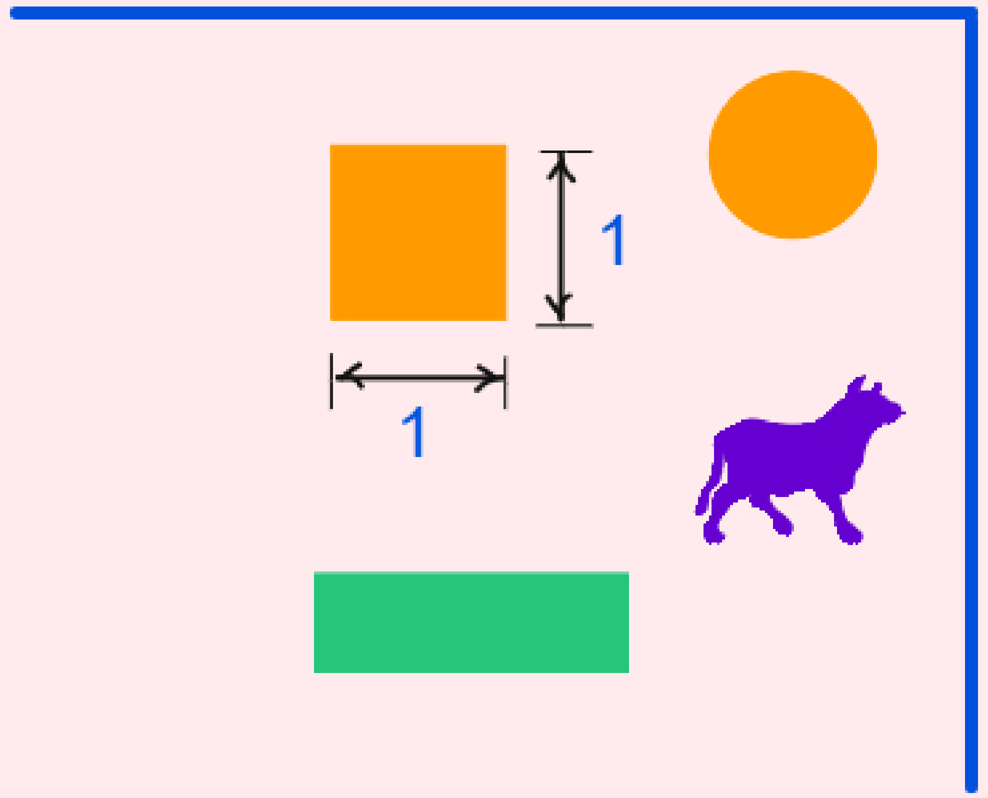
Representations for multiplication





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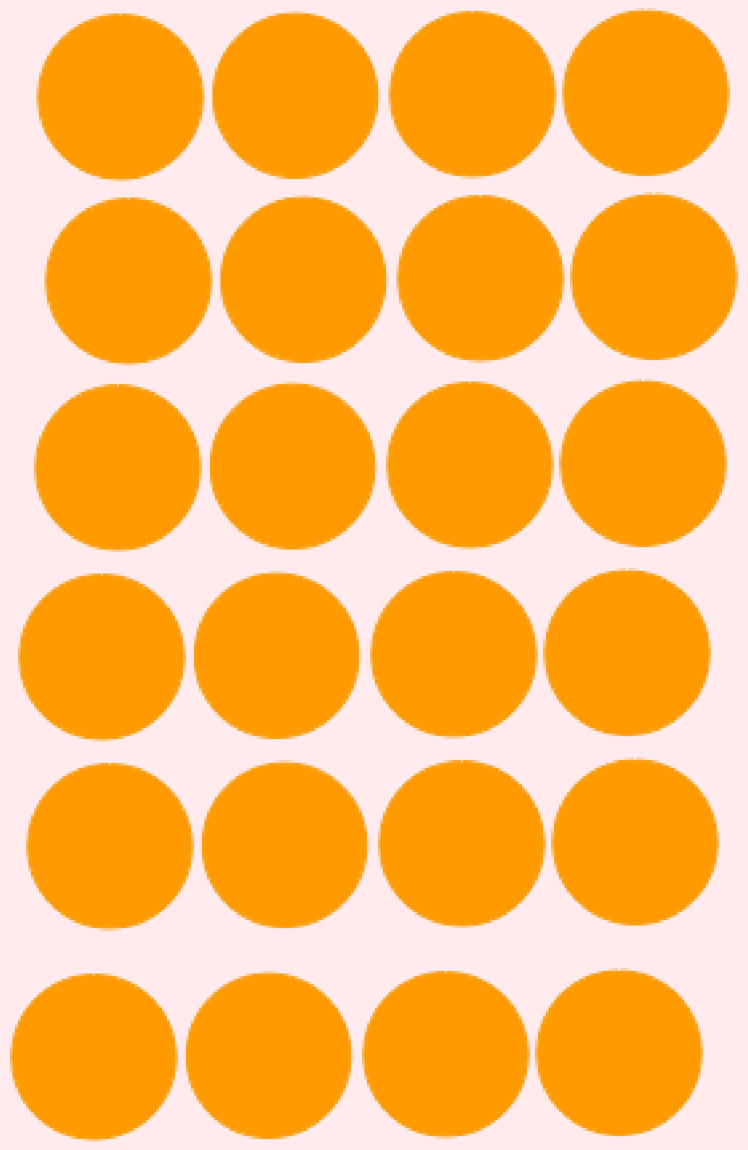
Associative property of multiplication

To "associate" or group together

$$(2\times3)\times4$$



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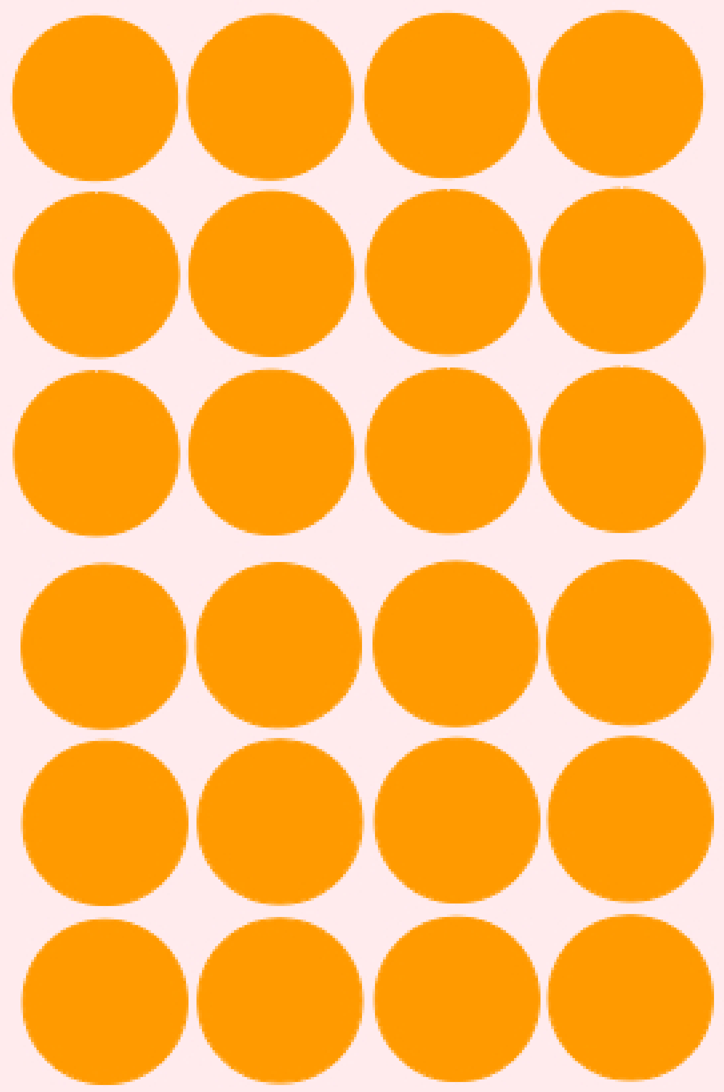
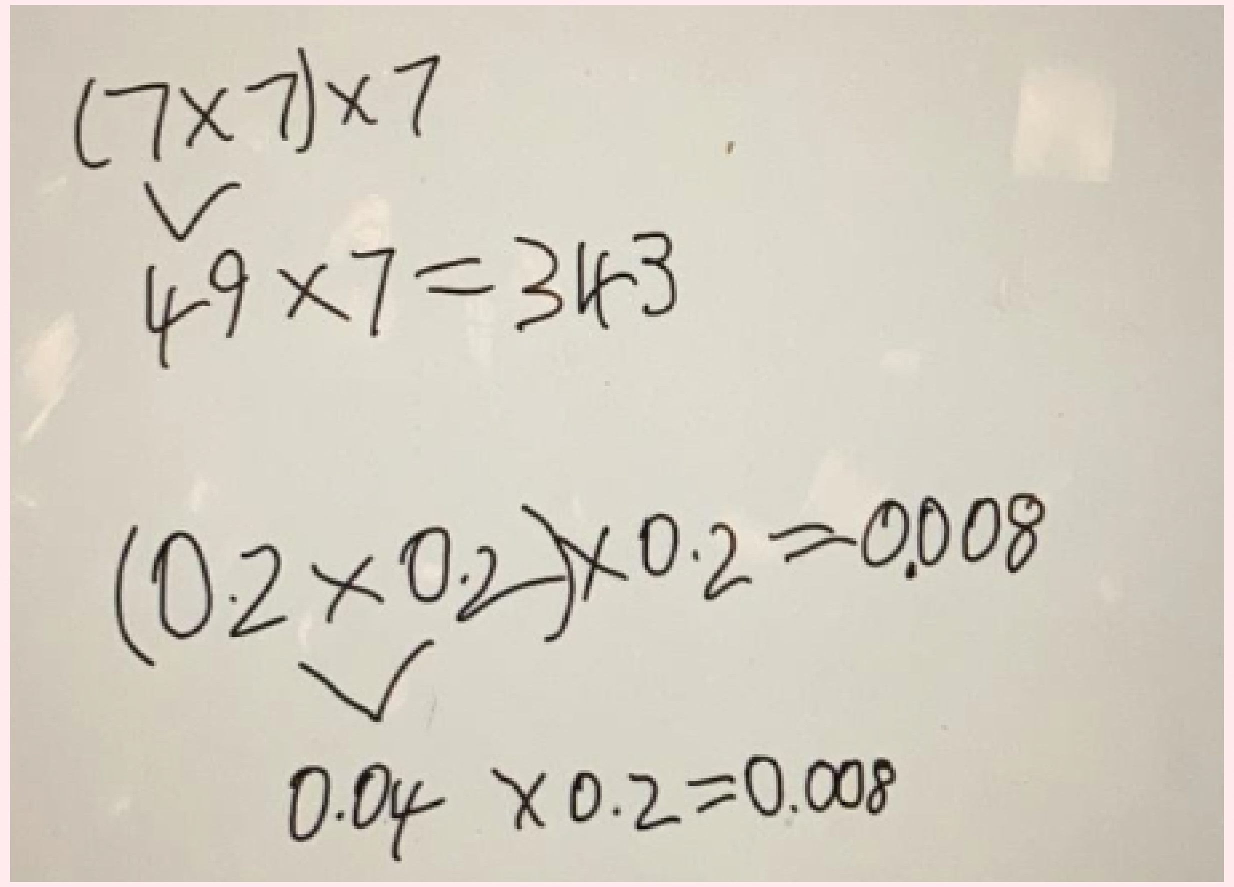


Associative property of multiplication

Exponentiation

$$2^3 = 2\times2\times2$$

$$(2\times2)\times2 \qquad 2\times(2\times2)$$



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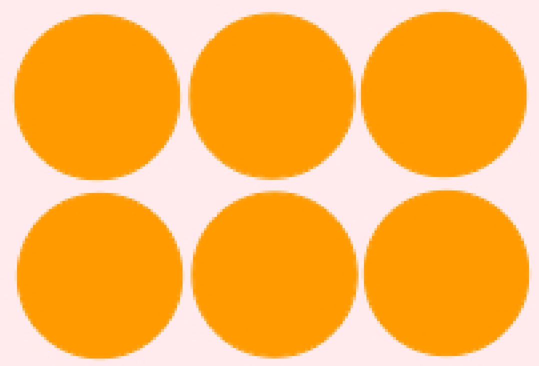
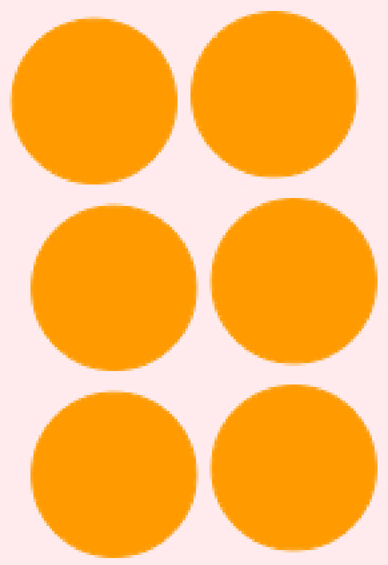
Commutative property of multiplication

2×3



3×2



$$3x^2yz$$

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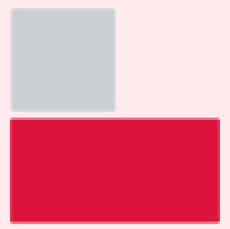
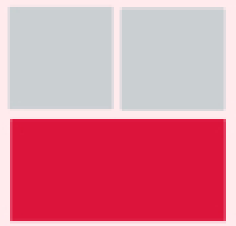
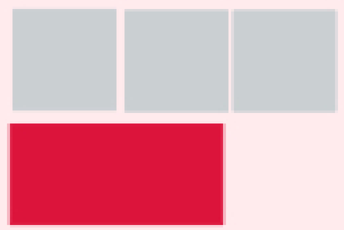


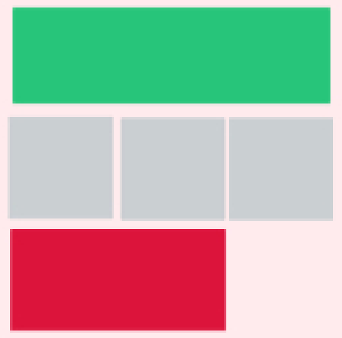
Show me a rod one half of red



Show me another half of red
(i.e two halves of red)

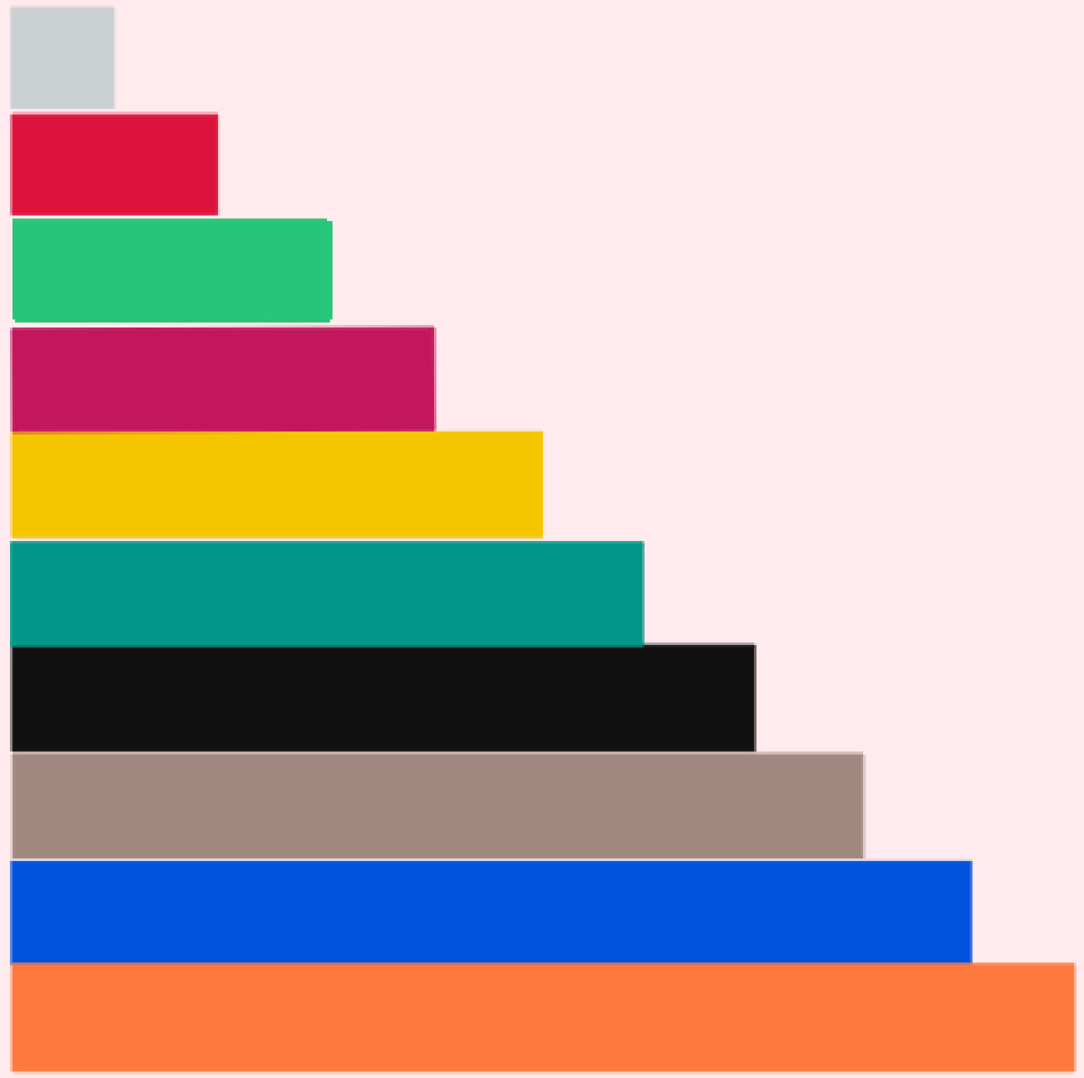


Show me another half of red
(i.e three halves of red)

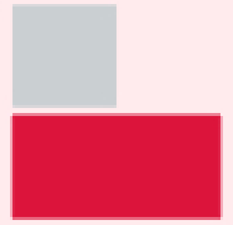


$$g = \frac{3}{2}r$$

Not a number on its own
Relational context

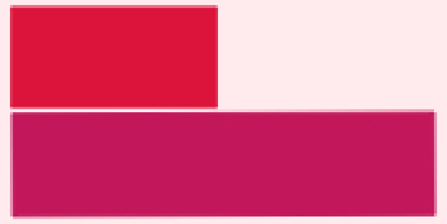


'Halfness'



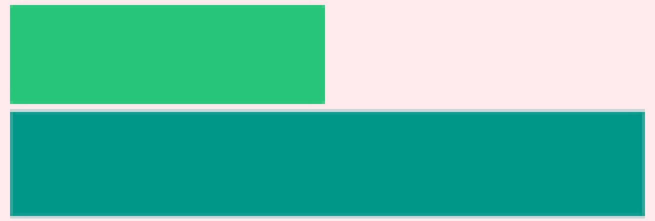
white = one half of red

$$w = \frac{1}{2}r$$



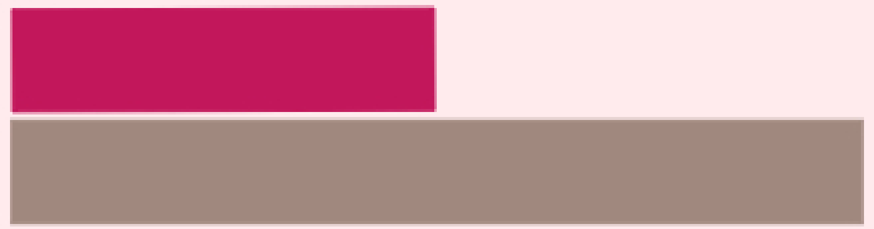
red = one half of purple

$$r = \frac{1}{2}p$$



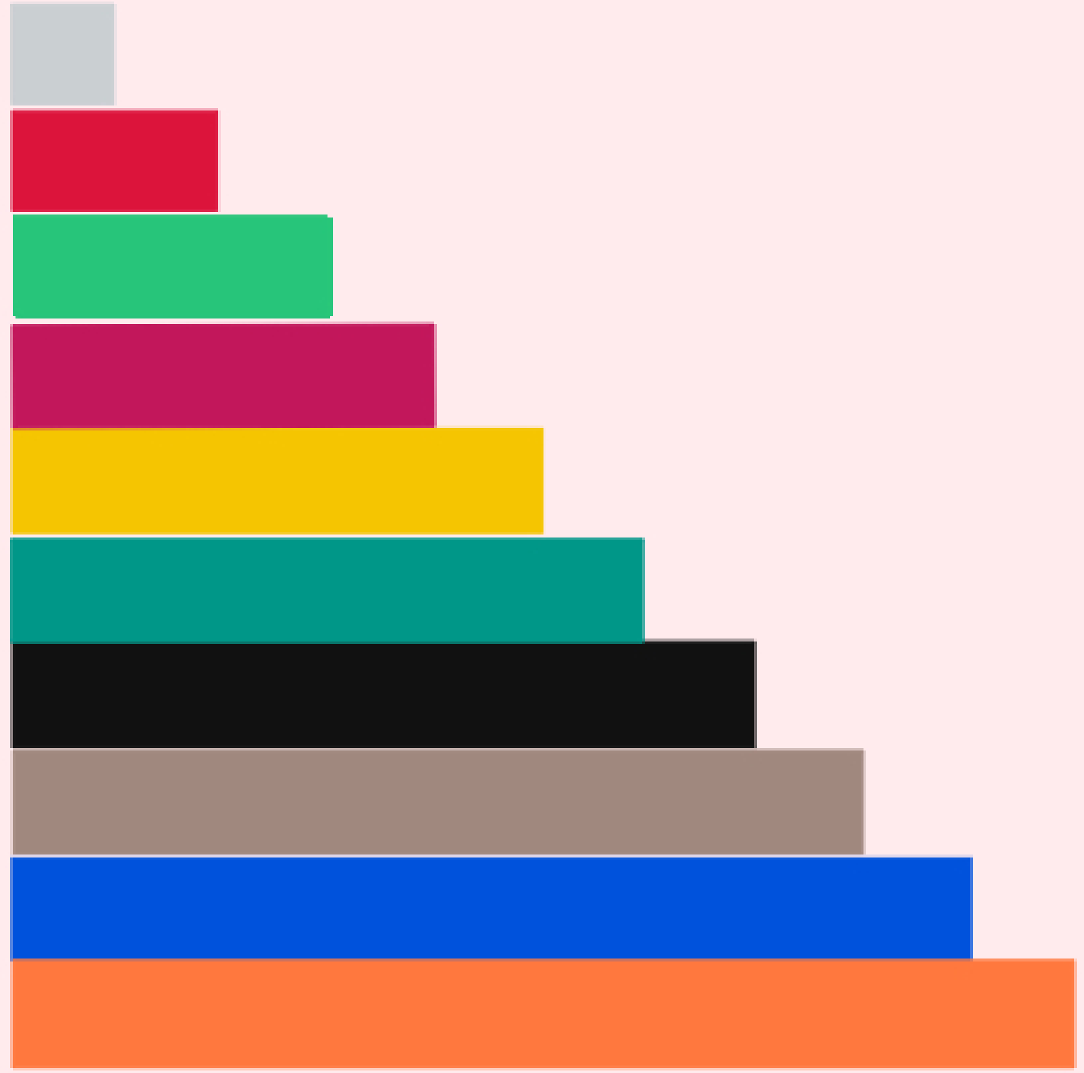
light green = one half of dark green

$$lg = \frac{1}{2}dg$$

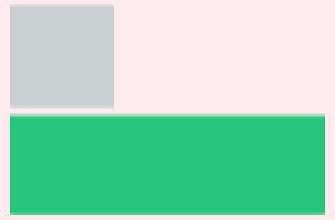


purple = one half of brown

$$p = \frac{1}{2}br$$



'Thirdness'



white = one third of light green

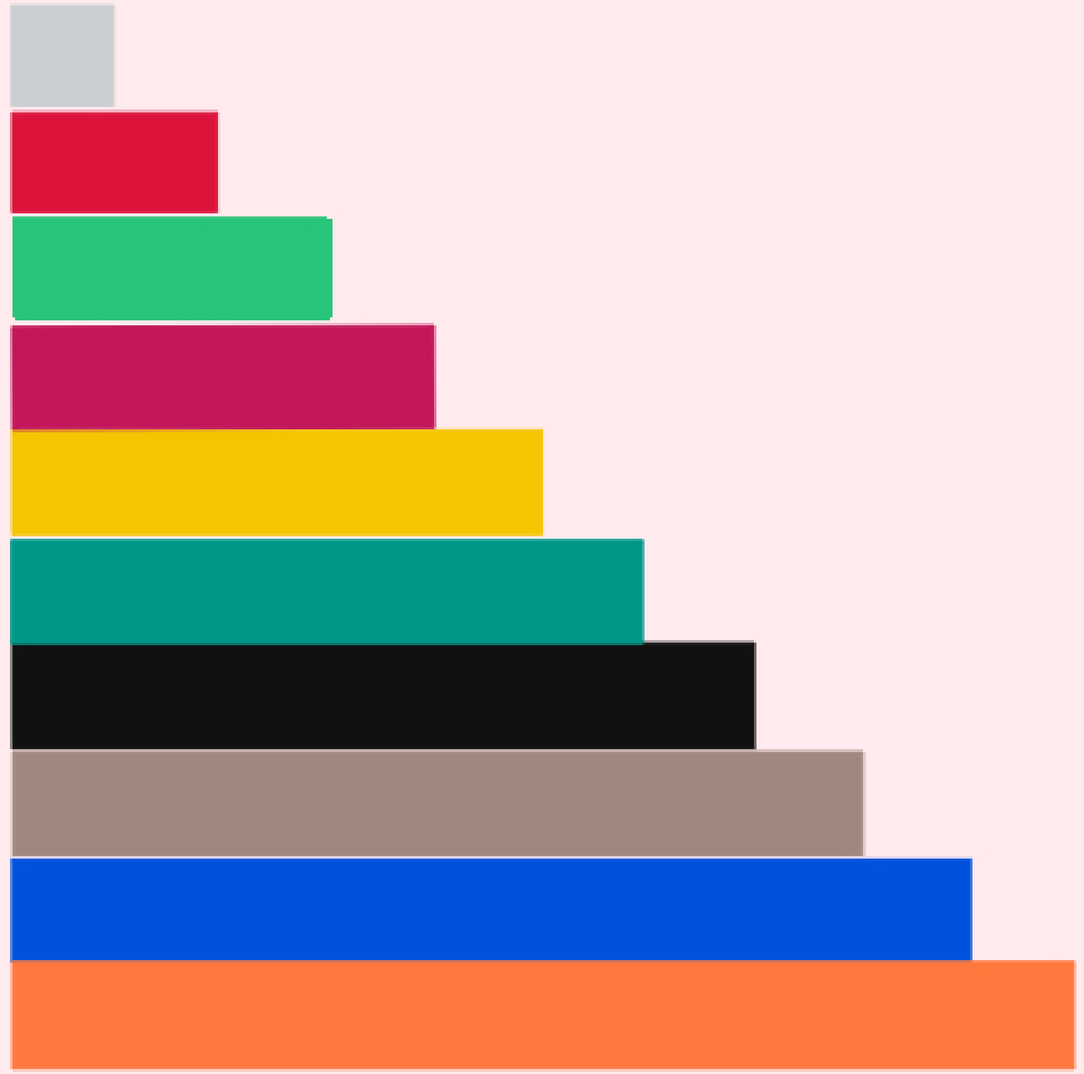
$$w = \frac{1}{3} lg$$

red = one third of dark green

$$r = \frac{1}{3} dg$$

light green = one third of blue

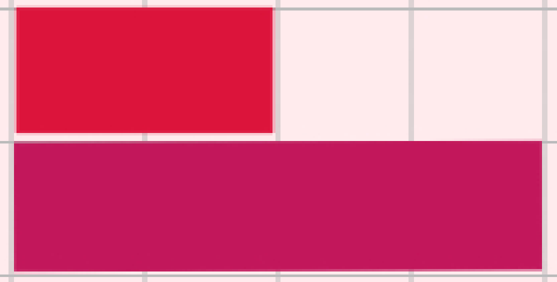
$$lg = \frac{1}{3} blu$$



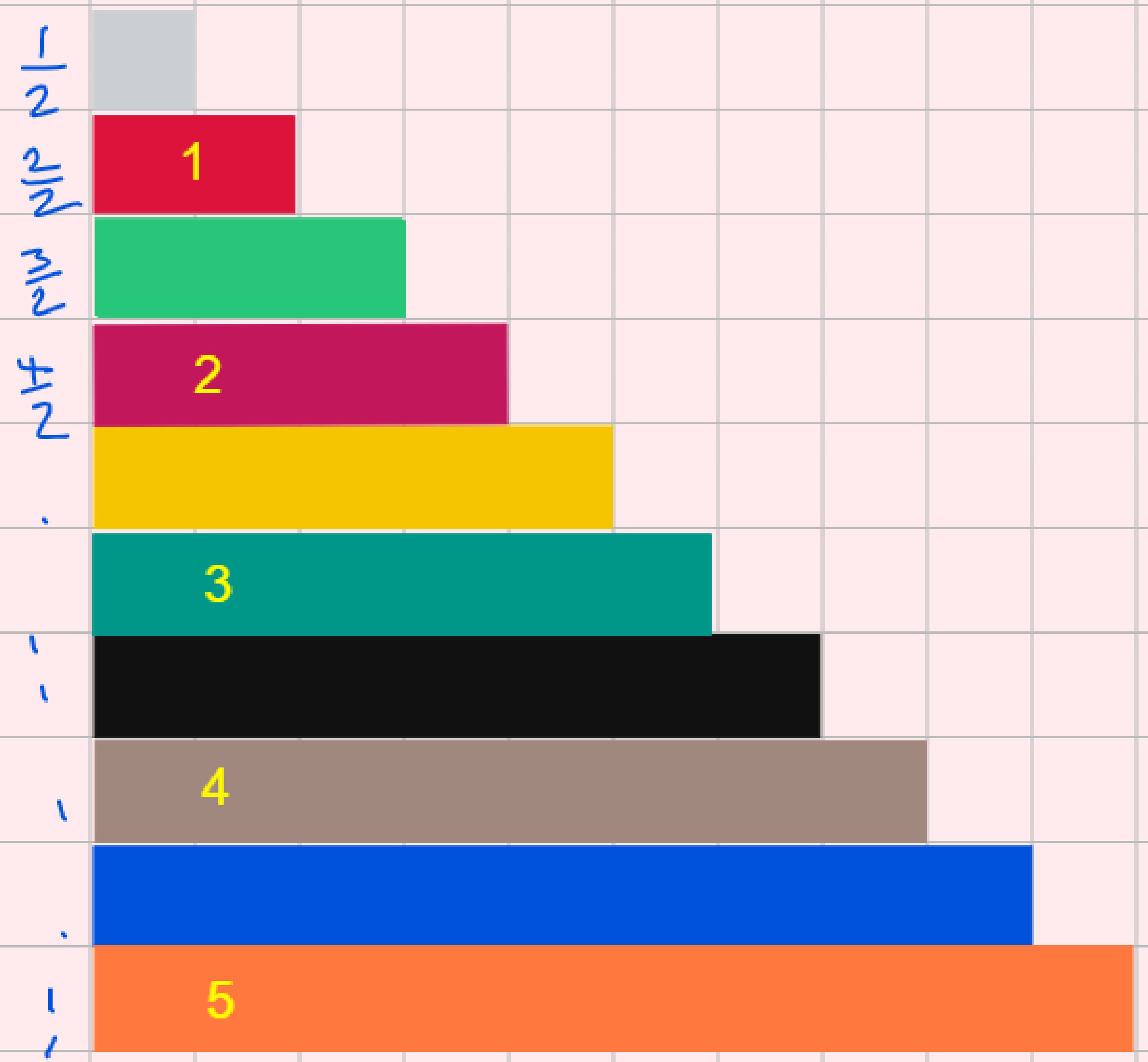
purple = one third of orange + red

$$p = \frac{1}{3}(o + r)$$

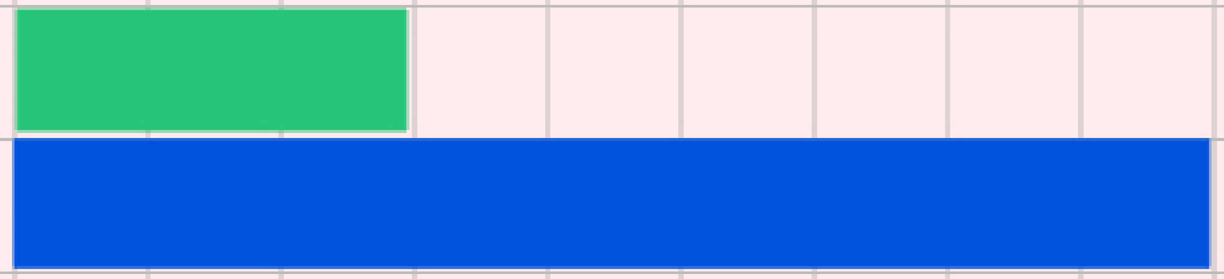
$$\frac{1}{2}p = r$$



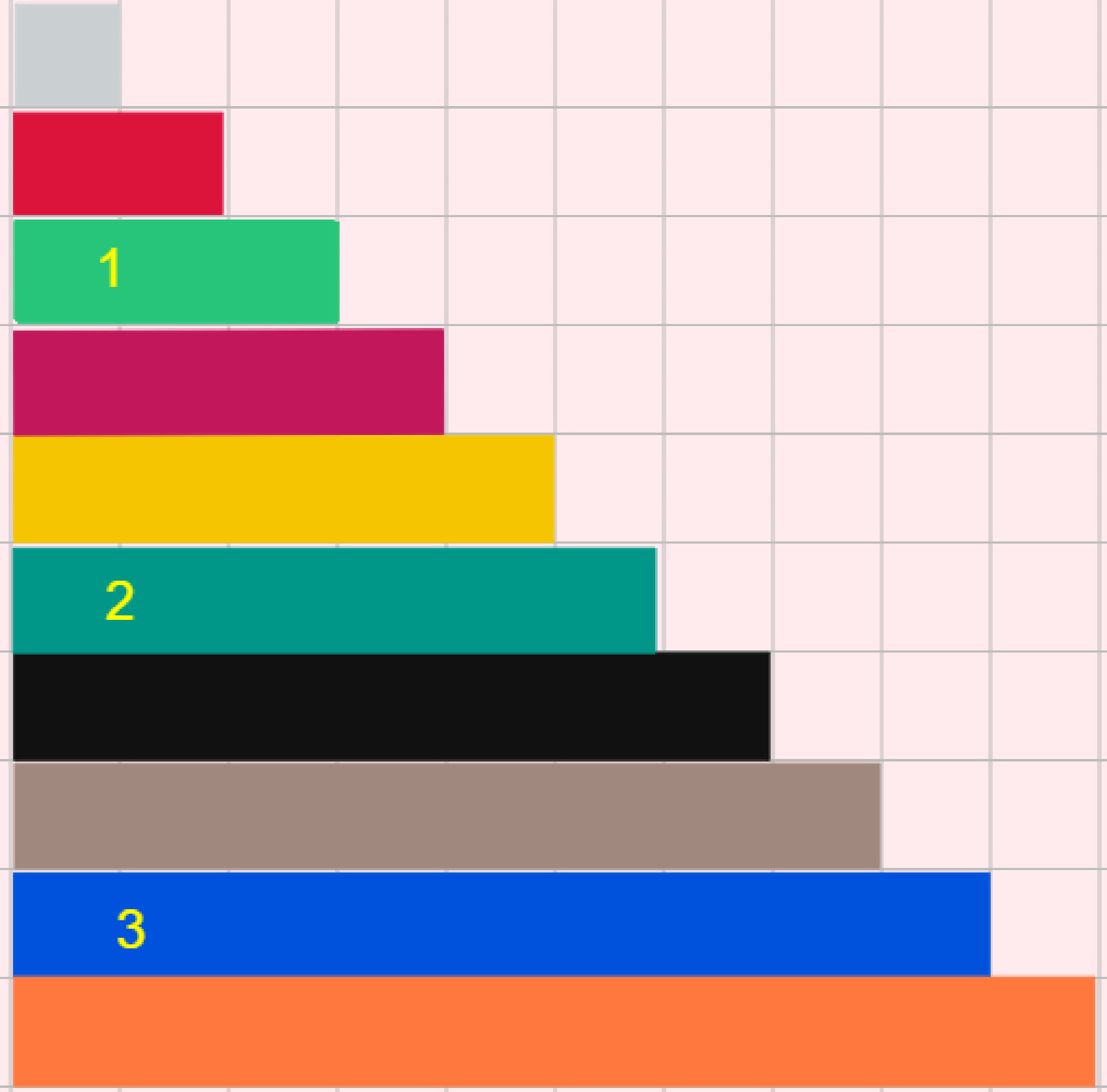
$$\frac{1}{2} \times 2 = 1$$

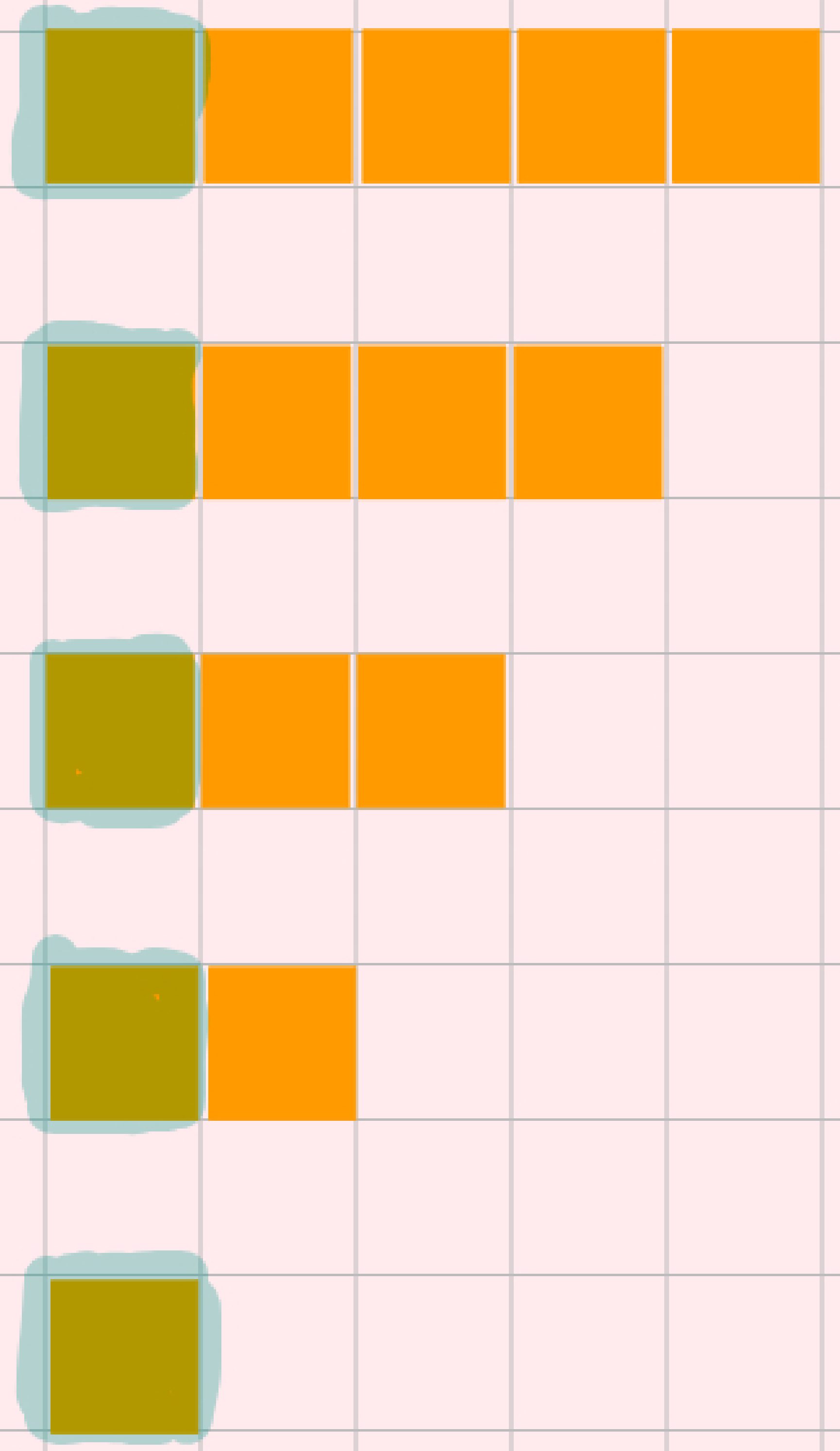


$$\frac{1}{3} blu = lg$$



$$\frac{1}{3} \times 3 = 1$$





$$\frac{1}{5}\times 5 = 1$$

$$\frac{1}{4}\times 4 = 1$$

$$\frac{1}{3}\times 3 = 1$$

$$\frac{1}{2}\times 2 = 1$$

$$\frac{1}{1}\times 1 = 1$$

$$\frac{1}{anything}\times anything = 1 \qquad (anything \neq 0)$$

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(Equality)

(Existence of multiplicative inverses)

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(Multiplicative identity property of 1)

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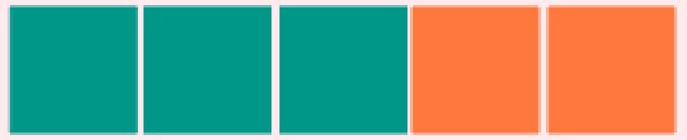
'Once you know the 1 of something'

1% of 400

3:2

1 part in a ratio 

$\frac{1}{5}$ of 400

The ingredients for cooking 1 pancake

Change in y for every 1 unit change in x

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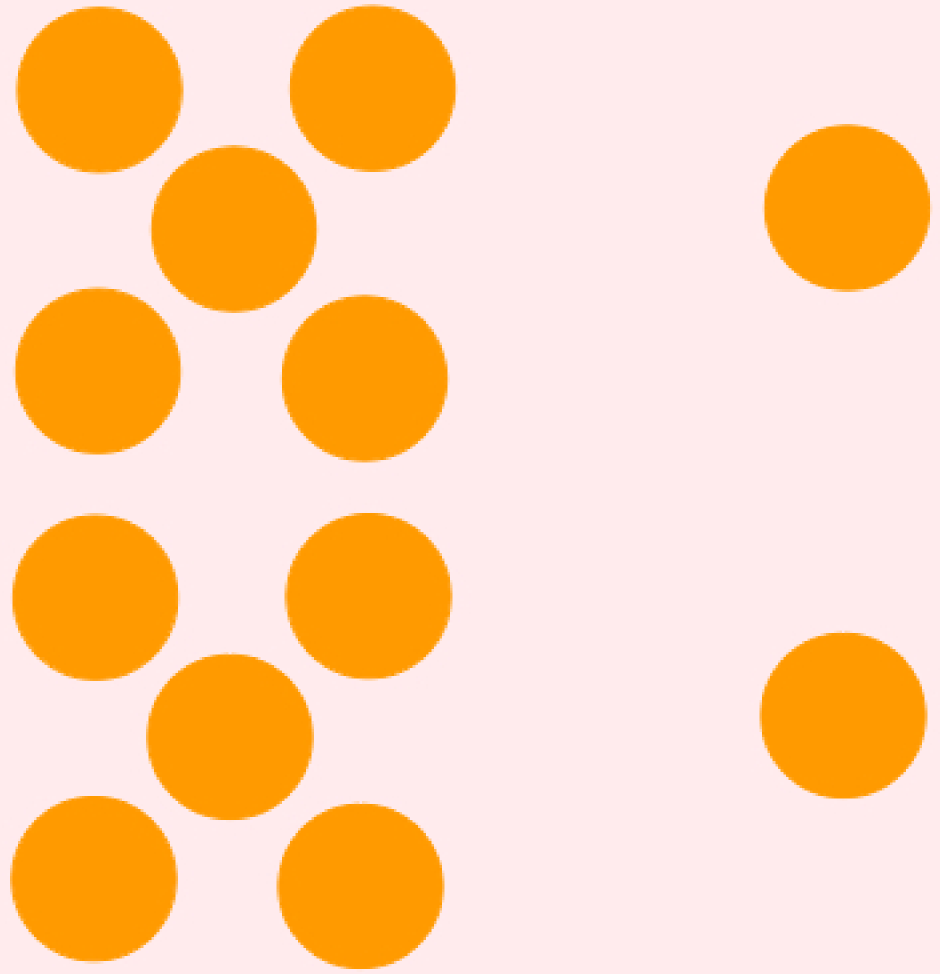
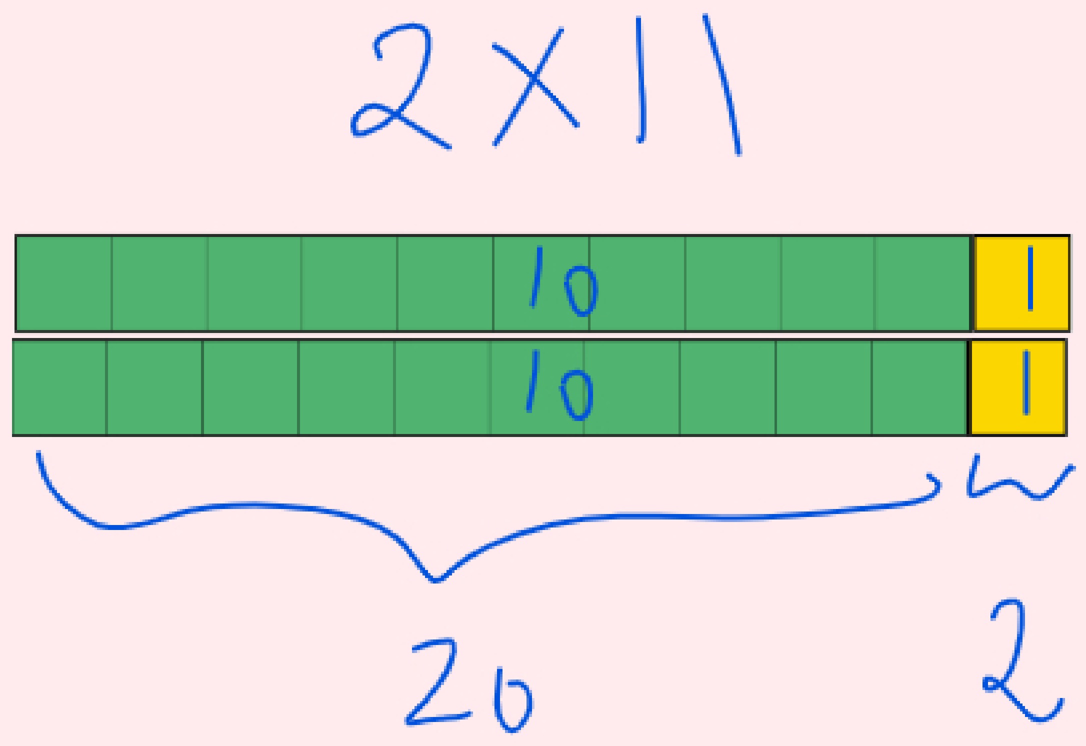
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	$a\times\frac{1}{a}=\frac{1}{a}\times a=1 \quad (a\neq 0)$
Distributive property of multiplication over addition	$a\times(b+c)=(a\times b)+(a\times c)$ $a(b+c)=ab+ac$

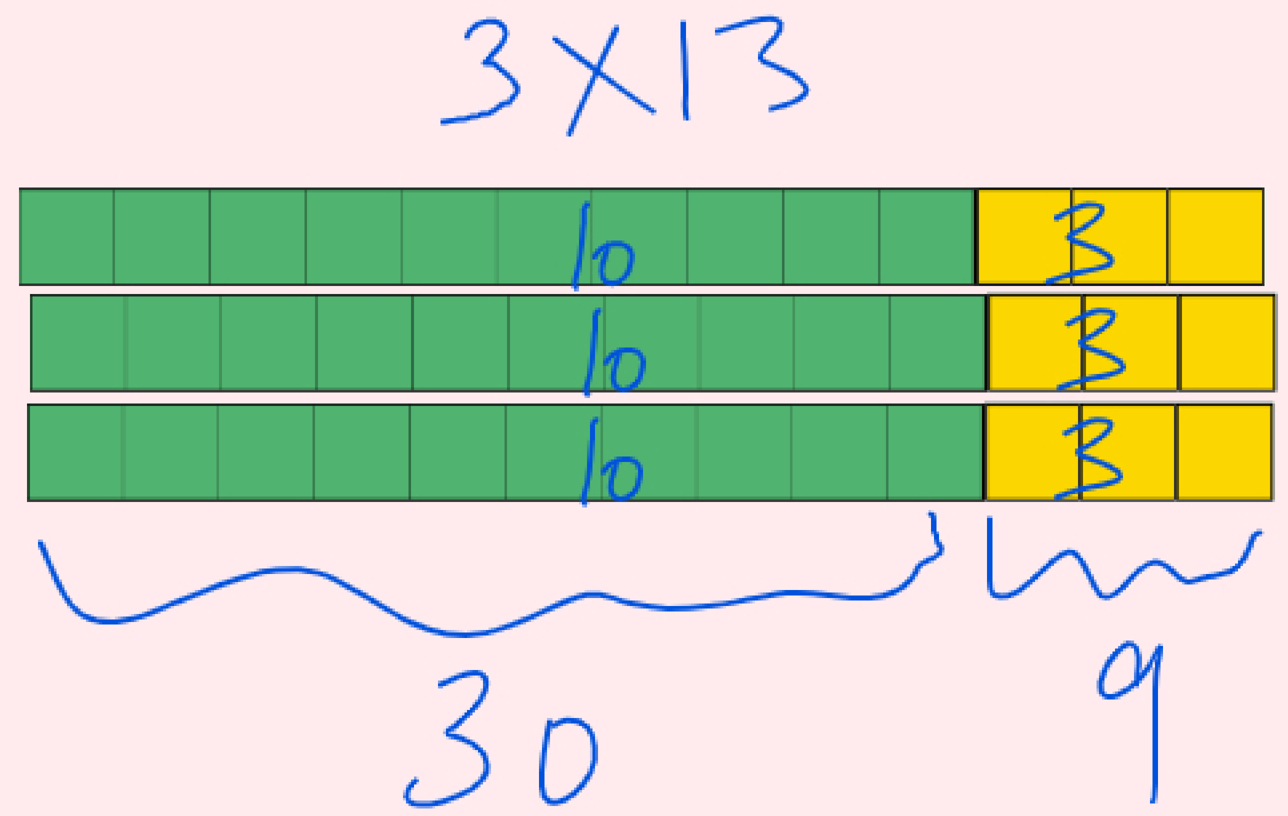
Distributive property of multiplication over addition

Conceptual Subitising

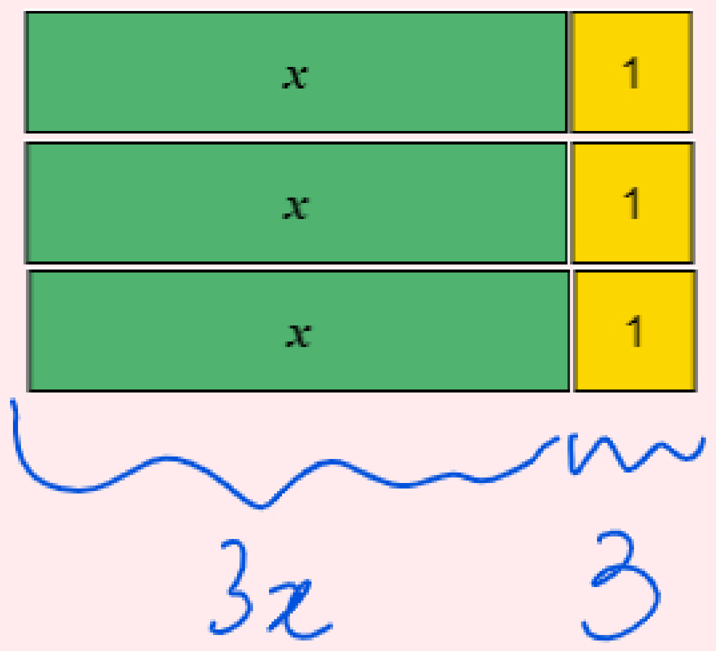


Matt Swain and Christopher Such
Making Friends with Numbers
#MathsConfMini


2 X 11
10
1
10
1
20
2


3 X 13
10
3
10
3
10
3
30
9

$3(x+1)$

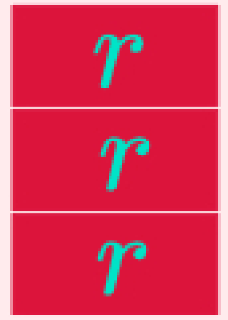


$4(-x + y + -1)$

$-4x + 4y + -4$

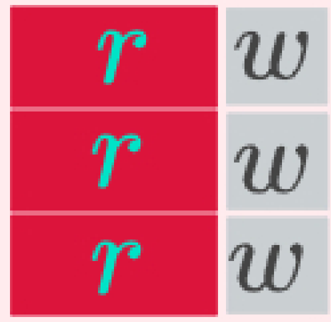
$-4x + 4y - 4$

Distributive Axiom

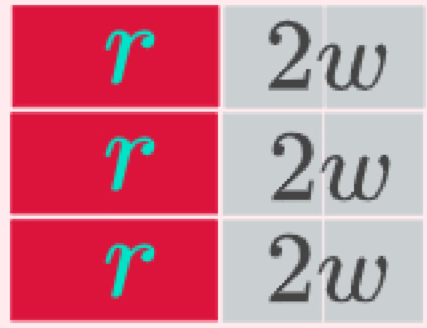


3 multiples of red

3r

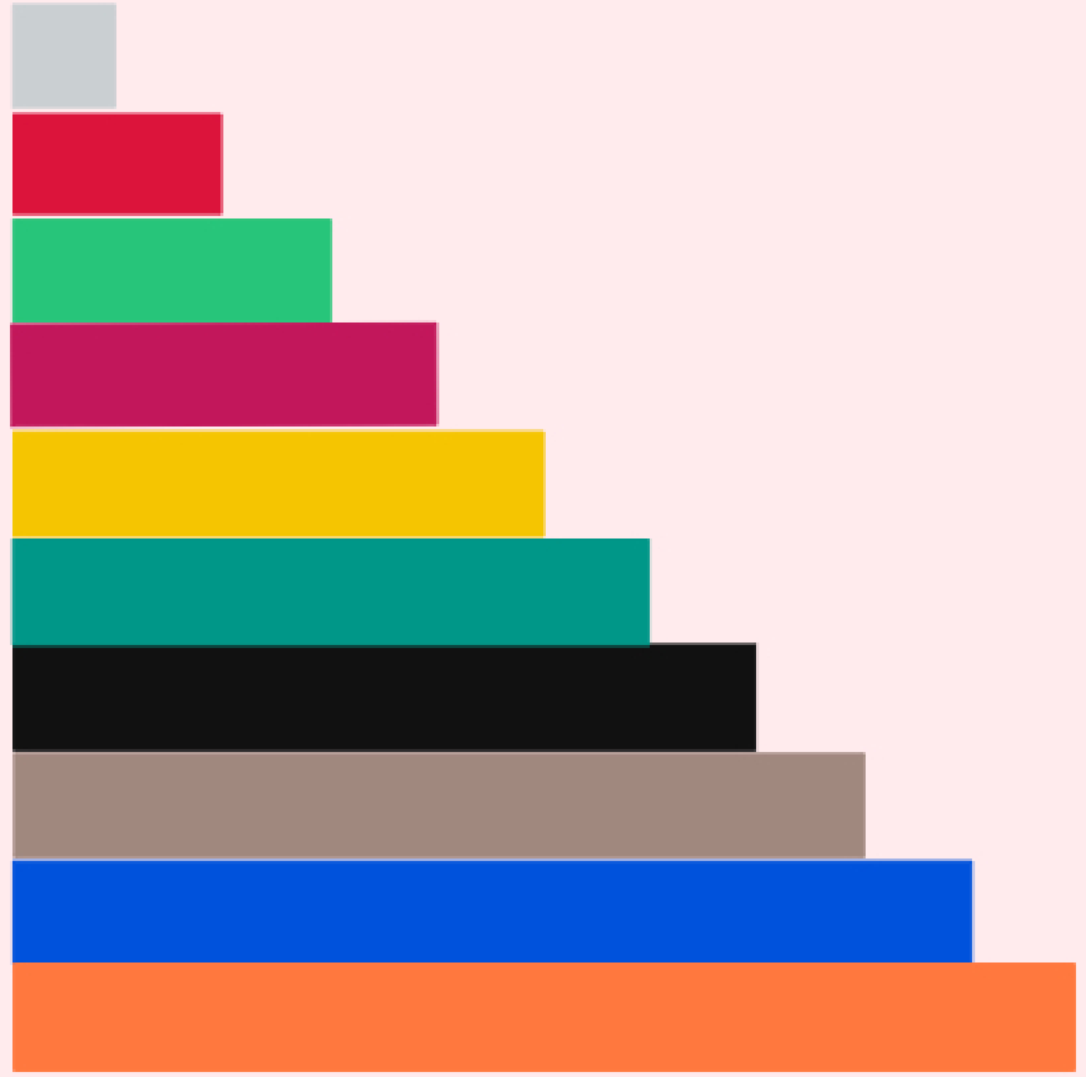


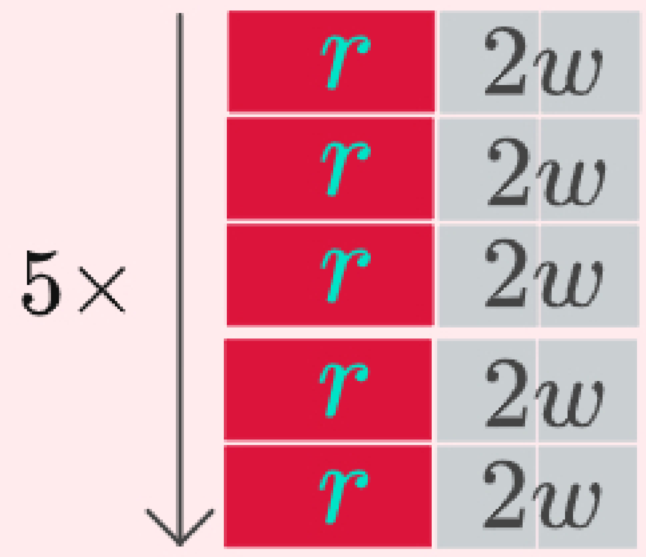
3 multiples of (red + white)
3(r + w)
3r + 3w



3 multiples of (red + two whites)
3(r + 2w)
3(r) + 3(2w)

3r+6w





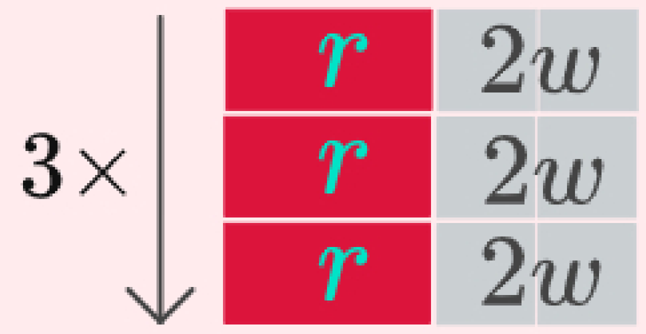
$(3+2)(r+2w)$

$5(r+2w)$

$5r+5(2w)$

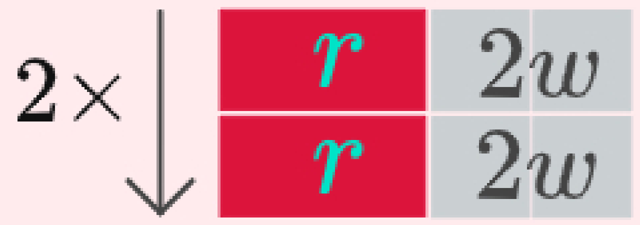
$5r+10w$

$(3+2)(r+2w)$



$3(r+2w)$

$3r+3(2w)$



$2(r+2w)$

$2r+2(2w)$

$$\begin{array}{r} 3r+6w \\ +\ 2r+4w \\ \hline 5r+10w \end{array}$$

$$(a+b)(c+d) = a(c+d) + b(c+d)$$

$$(5x+2)(x+3) = 5x(x+3) + 2(x+3)$$

$$(x+2)(3x^2+5x+4) = x(3x^2+5x+4) + 2(3x^2+5x+4)$$

BitPaper
Share Knowledge

Task Two

<u>Example</u>
Two possible attempts at "factorising" 80 are shown.
Each time a different factor has been used outside of the brackets.

80	80
= 8 x 10	= 5 x 16
= 8(4 + 6)	= 5(10 + 6)
= 8 x 4 + 8 x 6	= 5 x 10 + 5 x 6

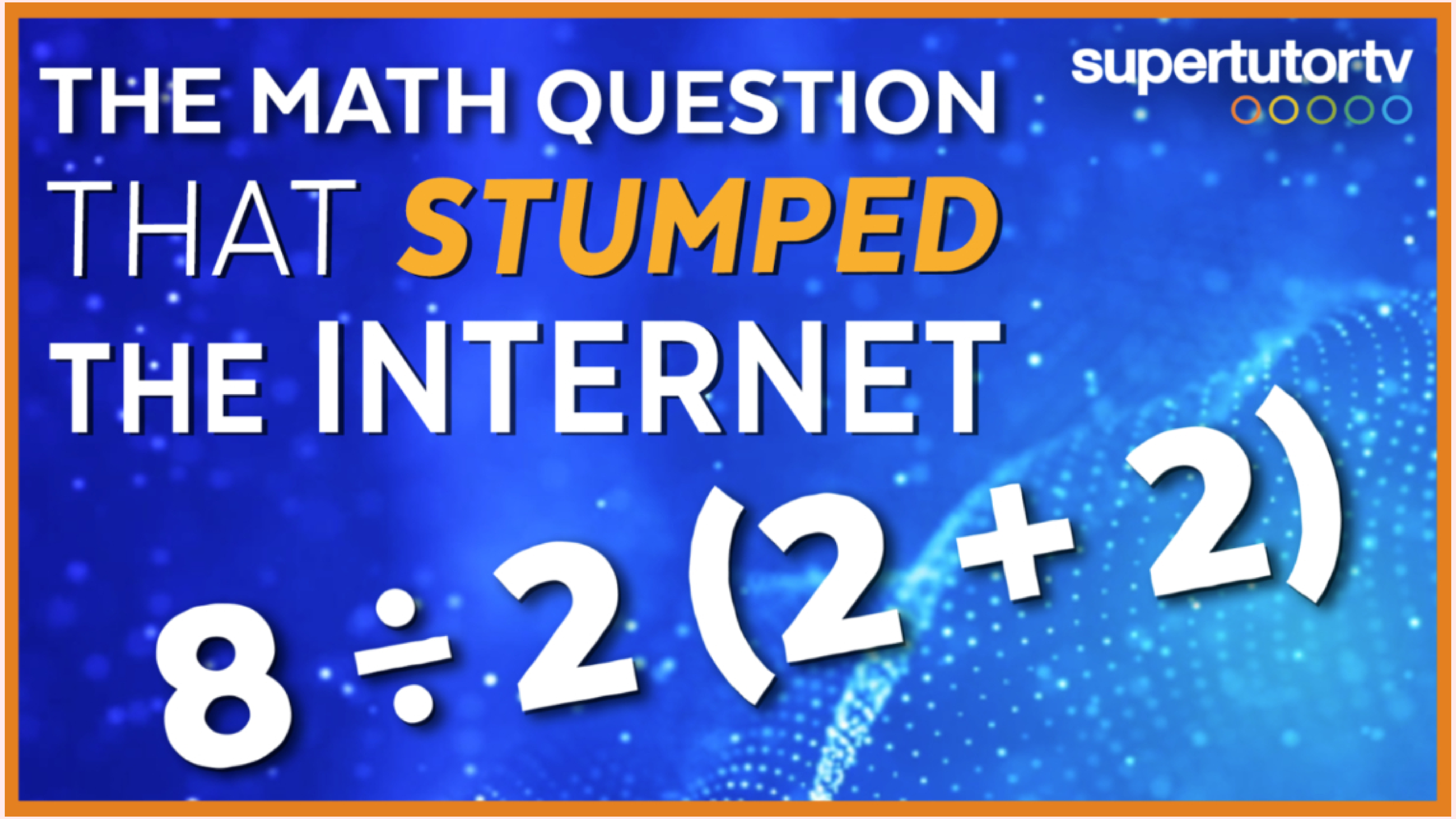
(a) Create two more factorisations of 80.

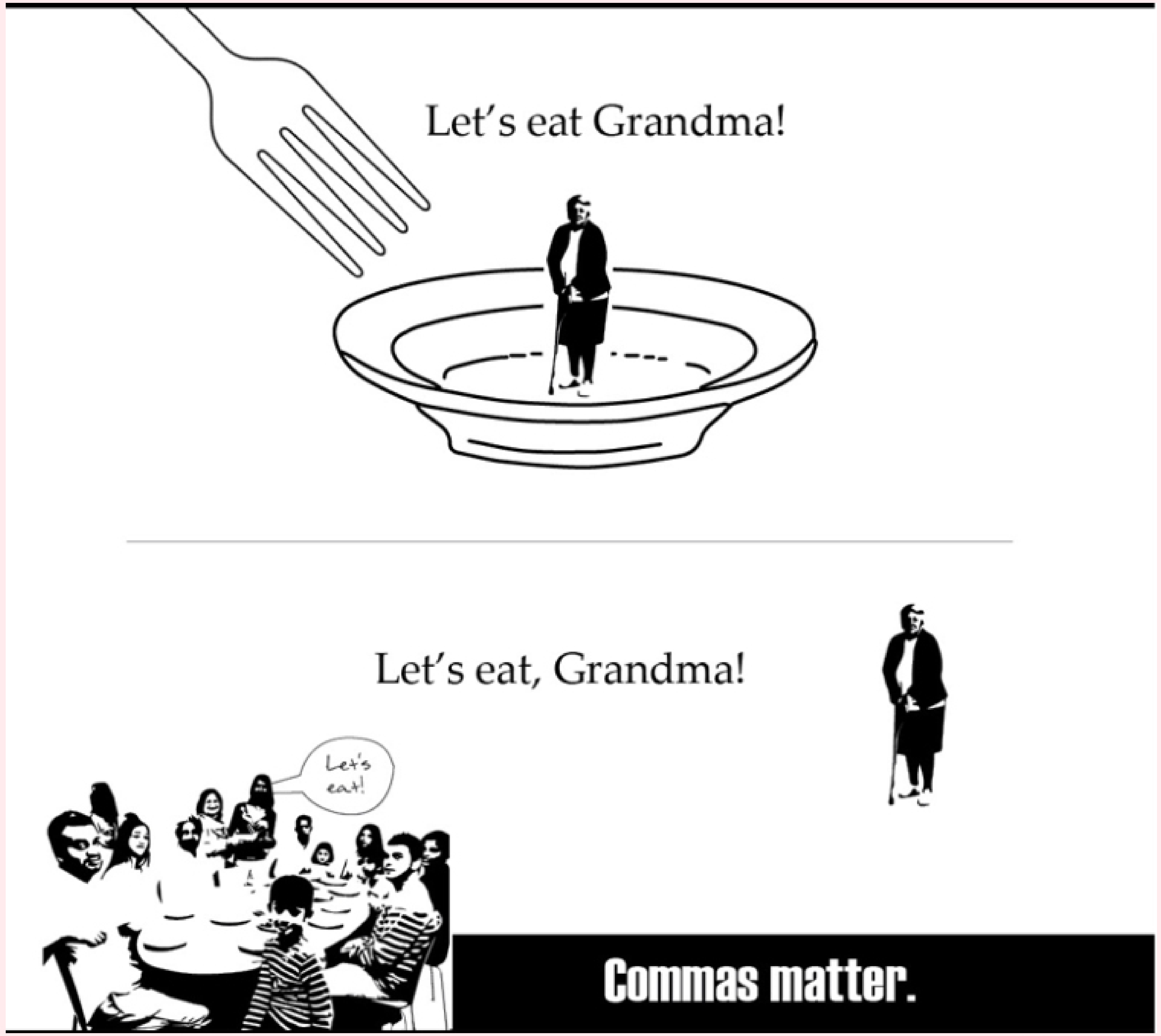
(b) Create three factorisations of 60.

(c) Create a factorisation of 10q

http://chrismcgrane.blogspot.com/2019/05/alg.html
A Progression from Long Multiplication to Algebraic Long Division

BIDMAS/PEMDAS...





https://digitalsynopsis.com/tools/punctuation-marks-importance-rules-usage/

$$s = vt - \frac{1}{2}at^2$$

$v = 5\ m/s$

$t = 2\ s$

$a = 1\ m/s^2$

$$5\times 2 - \frac{1}{2}\times 1\times 2^2$$

$$2+2+2+2+2-\left(\frac{1}{2}\times 1\times 2\times 2\right)$$

$$2+2+2+2+2-(1\times 1\times 2)$$

$$2+2+2+2+2-(2)$$

$$2+2+2+2+2+(-2)$$

BIDMAS

B A A A

Atul Rana

The Field Axioms: A reference framework of coherence

The Complete Mathematics Conference